Exercise 5

Prove that multiplication of complex numbers is commutative, as stated at the beginning of Sec. 2.

Solution

Here we have to show that

$$z_1 z_2 = z_2 z_1,$$

where z_1 and z_2 are complex numbers. Let $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$ and assume that x_1, x_2, y_1, and y_2 are real numbers.

$$\begin{aligned} z_1 z_2 &= (x_1, y_1)(x_2, y_2) = (x_1 x_2 - y_1 y_2, y_1 x_2 + x_1 y_2) \\ z_2 z_1 &= (x_2, y_2)(x_1, y_1) = (x_2 x_1 - y_2 y_1, y_2 x_1 + x_2 y_1) \end{aligned}$$

Because $x_1 x_2 - y_1 y_2 = x_2 x_1 - y_2 y_1$ and $y_1 x_2 + x_1 y_2 = y_2 x_1 + x_2 y_1$, the real and imaginary components of $z_1 z_2$ and $z_2 z_1$ are the same. Therefore,

$$z_1 z_2 = z_2 z_1.$$